

Harmonic Functionalism in Russian Music Theory: A Primer

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It is often noted that Russian music theory relies heavily on the theories of Hugo Riemann. At the same time, it is also noted that those of Heinrich Schenker have had virtually no currency in Russia. This should come as no surprise insofar as Schenker was such an inveterate anticommunist and anti-Marxist. Perhaps because of Russia's Schenkerian void, the history of step theory (*Stufentheorie*), in which Schenker serves as something of a culmination, rarely gets told. Yet Russia has a rich and long tradition with step theory, one that has always been part of its music theory. In fact, as I show below, many significant Russian music-theoretical works represent a hybrid of the two main harmonic systems that emerged from 18th and 19th-Century European practices, step theory and function theory (*Funktionstheorie*).

Riemann is generally considered the most well-known figure in the history of function theory—after all, it was he who took the word “function,” from mathematics, and applied it to musical chords. But his work did not arise in a vacuum of course. There were important precursors to his work. The story of how Riemann's work made it to Russia is generally underexplored, with the outstanding work of Ellon Carpenter in the 1980s the exception.¹ In examining harmonic functionalism in Russian music theory here, I follow in her footsteps.

I will look at three main figures in my music-theoretical primer: Nikolai Rimsky-Korsakov, Georgy Catoire, and Yuri Tiulin. Each of these figures produced noteworthy textbooks that help tell the story of Russian harmonic functionalism. Rimsky-

¹ Ellon Carpenter, “The Theory of Music in Russia and the Soviet Union, ca. 1650–1950.” PhD diss., University of Pennsylvania, 1988.

Korsakov's *Harmony Textbook* was the first to group together families of chords based on common-tone relationship, which he did eight years before Riemann. Catoire's *Theoretical Course of Harmony*, was the first to use the Riemannian designations T, D, and S in conjunction with harmonic analysis. While Yuri Tiulin, drawing on the work on Riemann as well as that of his countryman Boleslav Yavorsky, espoused a system that can rightly be called a hybrid system in his *Study of Harmony*.

Function Theory and Nikolai Rimsky-Korsakov

Despite its foundational nature with respect to our understanding of tonal harmony, the term “function” appears quite rarely in Riemann's *Vereinfachte Harmonielehre, oder Die Lehre von den tonalen Funktionen der Akkorde* (Hyer, 92). But at the end of the Preface Riemann gives one of his most important clues for what he means by “function”: “There are only three kinds of tonal functions [Funktionen] in harmony (meanings [Bedeutungen] within the key), namely that of the tonic, dominant, and subdominant” (cited in Hyer, 92).² In interpreting Riemann's words, Brian Hyer says that “it becomes clear that the notion of a tonal function refers either to chords or properties of chords..., and that there are three of them, coinciding with the tonic, dominant, and subdominant” (92–93). From this we music theorists, in both Europe and North America at least, have the famous T, D, and S formulations that are common in the music theory classroom.³

Of course, T, D, and S form the basis of function theory, which arose in contradistinction to the older step theory, with Jean Philippe Rameau as something of a progenitor to both. If Heinrich Schenker represents the culmination of step theory (with Georg Joseph Vogler, Gottfried Weber, Johann Kirnberger, and Simon Sechter as important predecessors), then Riemann represents the culmination of function theory (with Moritz Hauptmann and Arthur von Oettingen as important predecessors).⁴ One

² Here is the original German: *Es giebt nur dreierlei tonale Funktionen der Harmonie (Bedeutungen innerhalb der Tonart), nämlich die der Tonika, Dominante und Subdominante. In der Veränderung dieser Funktionen beruht das Wesen der Modulation* (Riemann 1893, 9). In general I place Russian, German, and French titles and text in italics in order to set them apart from the surrounding English text.

³ Many American textbooks substitute “PD” (pre-dominant) for “S,” which still leaves the tripartite nature of chord division intact. Generally, this tripartite nature is far more subdued in American music theory textbooks, which are based primarily on the theories of Heinrich Schenker.

⁴ For more on the history of step theory, and Schenkerian studies, in Russia, see Ewell 2018.

element often lost on Western theorists is that Russian tonal theory, to a very large extent, represents a hybrid version of these two systems.

Rimsky-Korsakov (1844–1908), with his *Учебник гармонии* (Harmony textbook), is sometimes cited as one of the originators of harmonic functionalism in Russia. In a 1950 review of a new edition of Rimsky-Korsakov's textbook, one post-WWII Soviet author, Joseph Ryzhkin, even went so far as to claim that it was a mistake to credit Riemann for functionalism and that Rimsky-Korsakov should be given credit for doing so (Ryzhkin 1950, 108). This view was ultimately understood as revisionist, and Riemann is generally given the same credit in Russia as elsewhere for his role in the history of harmonic functionalism. Still, what Rimsky-Korsakov did in his textbook, some eight years before Riemann's *Vereinfachte Harmonielehre*, is noteworthy.

Rimsky-Korsakov's textbook was first published in lithograph form in 1885. Rimsky-Korsakov had a wonderful correspondence with Petr Tchaikovsky about his new text.⁵ Until Rimsky-Korsakov set out to write his book, Tchaikovsky's *Руководство к практическому изучению гармонии* (Guide to the practical study of harmony) ([1871] 1957) was preeminent in Russia.⁶ Ellon Carpenter identifies four main advances in Rimsky-Korsakov's text: "1) a different scale system, 2) an initial restriction to the three principal triads, 3) an emphasis on melodic harmonization and independent writing with no figured bass, and 4) an innovative approach to modulation, based on key relationships and a modulatory plan" (1988, 314). His "three principal triads" were, of course, I, IV, and V. One notices Rimsky-Korsakov's emphasis on harmonic thinking and harmonic progression from this quote—his avoidance of figured bass, as an antiquated practice in 1885, hews to this harmonic thinking. His addition of the "harmonic major," a major scale with $\flat\hat{6}$, yielded several new harmonies in major, most importantly the minor subdominant and the fully diminished vii^o. Though Rimsky-Korsakov is often cited as the source of the rather frequent harmonic

⁵ The correspondence between Tchaikovsky and Rimsky-Korsakov about Rimsky-Korsakov's textbook is fairly well-trodden territory. In English, for instance, see Carpenter (1988, 292 and 312–314) and Jackson (1996, 221–242), who also does a comparison of the harmony textbooks of both composers. And in Russian, see Berkov (14–16), among others. Rimsky-Korsakov had asked his friend for commentary on the new textbook. To see Tchaikovsky's scathing remarks thereon, which he had written in the margins of the lithograph copy that Rimsky-Korsakov had sent to him, see Tchaikovsky 1957 (226–249), and Tchaikovsky 1945. For more on Rimsky-Korsakov's harmonic practices see my "On Rimsky-Korsakov's False (Hexatonic) Progressions outside the Limits of a Tonality" (Ewell 2019).

⁶ Tchaikovsky went on to write an abridged version of his text, the *Краткий учебник гармонии* (Short textbook on harmony) ([1874] 1957). Both versions have appeared in English: see Tchaikovsky (1871) 1900, and Tchaikovsky and Liliya Shamazov (2014), respectively.

major in Russia, he himself cited his friend and colleague, Anatoli Liadov, who cited his teacher, Yuli Iogansen (Rimsky-Korsakov [1909] 1980, 202). In fact, this system of harmonic major, a major scale with a $b\hat{6}$, came from Iogansen's teacher for his time in Leipzig, Moritz Hauptmann, one of the most important pre-Riemannian figures in the history of function theory.⁷

Notably, Rimsky-Korsakov's explanation that the three main triads generate the scale, and not the other way around, a bedrock of function theory, drew Tchaikovsky's ire. Rimsky-Korsakov says:

The main triads of the major and minor mode—the tonic, on the first scale degree, the subdominant, on the fourth scale degree and the dominant, on the fifth scale degree—are the main basis of any harmonization, since in any tonality consisting of these three triads all notes of the scale are present.⁸ (Rimsky-Korsakov 1885, 15)

To which Tchaikovsky responded: “This does not explain anything, since the entire scale is also contained in the II, V, and VI chords” (Tchaikovsky 1957, 232–233).⁹

Function theory represents the idea that T, D, and S are families or groups of chords, and not just the three triads built on scale degrees I, IV, and V. In Chapter 2, “Dissonant Chords of the Dominant Harmony,”¹⁰ Rimsky-Korsakov includes *vii*^{o7} as one such harmony, saying, “seventh chords built on VII replace the dominant-seventh chord in harmonizations... and can therefore be counted as *dominant harmonies* (1960, 61; italics original).¹¹ Thus he is clearly thinking of a grouping of dominants here. In

⁷With thanks to Richard Taruskin for pointing this out to me and citing the relevant sources (see also Taruskin 2019). For more on Hauptmann's role in Russian music theory see Jackson 1996, 147–158.

⁸ *Главные трезвучия мажорного и минорного лада: тоническое—I ступени, субдоминантовое—IV ступени и доминантовое—V ступени – суть главная основа всякой гармонизации, так как в тонах, составляющих эти три трезвучия, заключаются все ступени гаммы.* The Russian word for “mode” in this quotation, *лад*, is extremely complicated. I explain why this is so in my “On the Russian Concept of *Lād*, 1830–1945” (Ewell, 2020a). I will generally use “*lād*” (noun) and “*lādov*” (adjective) in this article.

⁹ *Это ничего не объясняет, ибо в трезвучиях II, V и VI ступеней тоже вся гамма.*

¹⁰ *Диссонирующие аккорды доминантовой гармонии.*

¹¹ *Септаккорды VII ступени заменяют собой в гармонизации доминантсептаккорд...а потому могут быть причислены к доминантовой гармонии.*

the third chapter of his textbook, “Secondary Steps of the Major Mode,”¹² Rimsky-Korsakov explains these groupings. After mentioning how, until this point, he had only considered I, IV, and V, and VII, he brings in the remaining II, III, and VI chords. He comments on the common tones and the relative-key relationships between what he calls the “main” (*главные*) and “subordinate” (*побочные*) chords. Example 1, from Rimsky-Korsakov’s textbook, is how he shows these common tones and relative relations (I have added Riemannian designators below), after which he adds:

It is easy to notice that the relationship of the subordinate triads among themselves is the same as those of the main triads, and for this reason all six triads of the first six scale degrees can be divided, in terms of character, into three groups: 1) the *tonic group*—the triads of the I (major) and VI (minor) degrees; 2) the *subdominant group*—the triads of the IV (major) and II (minor) degrees; and 3) the *dominant group*—the triads of the V (major) and III (minor) degrees.¹³ (1960, 66; italics original)

Because of these groupings Rimsky-Korsakov’s theories have been called “incipiently functional” (Carpenter, 317). I believe that Rimsky-Korsakov’s chord grouping are more than just this but, rather, a significant first step in the theoretical literature toward a fully-fledged function theory. True, he had a simpler, and certainly more practical, type of functionality in mind when compared with Riemann, but Rimsky-Korsakov’s textbook remains a significant precursor to Riemannian functionality. Unfortunately, his work, both theoretical and compositional, remains woefully underexplored in the West, a point that Richard Taruskin has highlighted in his work (see especially Taruskin 2011). Finally, as with all textbooks in Russia in the late nineteenth century, all harmonies are notated with roman numerals in step-theory fashion—T, D, and S appeared only in theory, not in print.

¹² *Побочные ступени мажорного лада.*

¹³ *Легко заметить, что отношения побочных трезвучий между собою—то же самое, что и главных, а потому все трезвучия первых шести ступеней можно подразделить по характеру их на три группы: 1) Тоническая группа—трезвучия I (мажорн.) и VI (минорн.) ступеней. 2) Субдоминантовая группа—трезвучия IV (мажорн.) и II (минорн.) ступеней. 3) Доминантовая группа—трезвучия V (мажорн.) и III (минорн.) ступеней.*

Example 1: Rimsky-Korsakov, relations between I and VI, IV and II, and V and III (1960, 66) (Riemannian designators added below)

I VI IV II V III

 T Tp S Sp D Dp

Georgy Catoire

The first true proponent of Riemannian theory in Russia was Georgy Catoire (1861–1926) who, at the suggestion of Tchaikovsky, went to Berlin in 1885 to study piano and composition, ultimately with Otto Tiersch and Philipp Rüfer.¹⁴ After Catoire's return to Russia in 1887 he studied with Rimsky-Korsakov, among others, so it was only natural that he combine his experience in Germany with his studies under Rimsky-Korsakov.¹⁵ About Catoire's *Теоретический курс гармонии* (Theoretical course of harmony) (1924–1925), Carpenter writes:

The introduction to the theory of functional harmony in Catoire's textbook was unique among Russian textbooks of this time. Although Catoire invented neither the idea nor the method of its presentation, his adaptation of the principles of Riemann's theories...became a permanent part of the Soviet theory of harmony. (1988, 603)

The textbook is unlike anything before it.¹⁶ Instead of a practical manual laying out exercises for students to complete, in the vein of partimenti or composition manuals before it—and Rimsky-Korsakov's textbook—Catoire's work reads much more like a

¹⁴ I have been unable to find out whether Catoire came into contact with Riemann during his time in Germany.

¹⁵ Significantly, Catoire was also influenced by the Belgian François-Auguste Gevaert and his *Traité d'harmonie théorique et pratique*, from 1905–1907 (see Catoire, vol. 1, 2). Notably, I was unable to find any trace of Riemannian T, D, and S in Gevaert's treatise, but only extensive use of roman numerals. See Gevaert 1905–1907.

¹⁶ Carpenter gives an exhaustive, if at times confusing, account of Catoire's textbook in Carpenter 1988, 577–604. This material is also largely contained in Carpenter (1983) 2009, 273–292.

contemporary theory textbook, with numerous examples by composers such as Bach, Beethoven, Borodin, Brahms, Grieg, Liadov, Weber, and Wagner. In the Preface, Catoire explains the nature of his work, and explicitly uses the term “functional” and “theoretical” to set his work apart from those that came before, saying that he will,

... build a system that connects the existing harmonies in today’s music while explaining their genesis, their interconnectedness, and their *functional* meaning, while keeping an eye on the possibility of their further treatment. In contrast to the abovementioned textbooks it is possible to call such a course *theoretical*.¹⁷ (vol. 1, 1; first italics mine, second italics original)

Early in Catoire’s textbook he lays out the tripartite system for chords in a diatonic system. Under the heading “Formation of the Diatonic System,”¹⁸ Catoire explains a system in which the tonic is C and the dominant and subdominant are G (above) and F (below). He then writes:

From the juxtaposition of these three most closely harmonically related sonorities: [see Example 2], the *simplest tonal system* is created. In this system we will call C the *tonic*, G the *dominant*, and F the *subdominant*. We will designate the sonorities built on them in the following fashion: tonic with the letter “t,” dominant with the letter “d,” and subdominant with the letter “s.”¹⁹ (vol. 1, 4–5)

To my knowledge, this is the first time German T, D, and S appear in any Russian music theory textbook.²⁰ Catoire adopted these three letters from German, and from

¹⁷ ...построить систему, объединяющую существующие в нашей музыке аккорды, объяснить их происхождение, их взаимную связь, их функциональное значение и усмотреть возможность их дальнейшего обогащения. В противоположность вышеупомянутым учебникам такой курс можно назвать теоретическим.

¹⁸ Образование диатонической системы.

¹⁹ Из сопоставления таких трех ближайших по гармоническому родству созвучков: [see Example 2], создается простейшая тональная система. В ней мы будем называть C—тонику, G—доминантой, F—субдоминантой. Построенные на них созвучия будем обозначать: тонический через t, доминантный через d, субдоминантный через s.

²⁰ Catoire would use lowercase t, d, and s in order to designate the consonant triad and uppercase to designate a diatonic-seventh T, D, and S. For my primer, when I speak of harmonic functions in the abstract, I will stick with uppercase T, D, and S.

Riemann. Example 2 shows the three triads that make of the diatonic system in C major. Catoire opts for the word “sonority” (*созвук*) instead of “triad” (*трезвучие*), which was already quite common in 1924 Russia.²¹ I believe he does this in order to emphasize the idea that T, D, and S, are representative of more than just triads, which immediately evoke the idea of key. Rather, T, D, and S are more abstract than the triads that make them up—they represent sonorities, representatives of families of chords that can *function* as T, D, or S, in a given diatonic system.

Example 2: Sonorities of the Diatonic System (Catoire, vol. 1, 5)

C	G	D
A	E	H
F	C	G
“s”	“t”	“d”

Under the heading “Displacement of One of the Tones of the Sonority T,”²² Catoire further defines chord function. (There are separate headings for D and S.) Catoire says:

The displacement of the root tone of the sonority “t” downward or of the fifth upward creates newly constructed triads, the *minor sonorities* on scale degrees III and VI. Such a shift of one of the tones of “t” has an *unresolved and indecisive* character and does not lead us to the “d” and “s” sonorities. The triads on the III and VI scale degrees serve as *replacements of the tonic triad* and, as such, perform the function of *tonic chords*.²³ (Vol. 1, 15; italics original)

²¹ *Трезвучие* is a coinage from Modest Rezvoi, which he translated into Russian from German *Dreiklang* in 1830. For more on this see Ewell 2020a, [2.3–2.4].

²² *Смещение одного из тонов в созвукe т.*

²³ *Смещение основного тона созвуча t вниз или квинты его вверх создает новые по своему построению трезвучия: это—минорные созвучия III-ей и VI-ой ст. Такое отклонение одного из тонов t носит характер нерешительный, неопределенный и не приводит нас еще к*

At this point he gives an example that I have reconstructed in Example 3, which is quite similar to Rimsky-Korsakov's example (see Example 1). Further, he shows the common chords and, in the first two instances in Example 3, shows the most parsimonious voice leading, with only one voice moving, in typical Riemannian fashion. Catoire ends this section with an example that I have recreated in Example 4. Note the brackets he includes, indicating III and VI functioning as tonic triad. Notably, he has not labeled the bracketed chords as T in the example, but only explained this in the prose. In fairly short order he does begin labeling chords with T, D, and S, with some alterations and many additions.

Example 3: Sonorities of the Diatonic System (Catoire, vol. 1, 15)

Example 4: III and VI Functioning as Tonic (Catoire, vol. 1, 15)

Importantly, Catoire envisioned a 10-note system in which he freely used the three flatted scale degrees from the natural minor, flat $\hat{3}$, $\hat{6}$, and $\hat{7}$. Further, by forming new dominant-seventh chords in a major mode from those three flatted degrees, Catoire creates three new “subordinate tonalities”²⁴ (Vol. 1, 55). That is, in C major, F major

созвукам d и s. Трезвучия III-ей и VI-ой ст. служат заместителями тонического трезвучия и как таковые исполняют функцию тонических аккордов.

²⁴ *Подчиненный строй*. For more on the use of “stroj” (*строй*) as “tonality,” see Ewell 2020a, [2.12].

(IV) is a new tonality tonicized by the dominant seventh of F (C dominant seventh, with the B^b, b[♯]). Similarly, B^b major (bVII, tonicized by F dominant seventh, with E^b) and E^b major (bIII, tonicized by B^b dominant seventh, with B^b and A^b) are new “subordinate” tonalities.

Under the heading “Dissonant Chords of the Major-Minor System,”²⁵ he lays out all possibilities for dissonant chords. I have shown this expansion of the system in Example 5. In Example 5a, Catoire shows two types of dominant chord. Superscript “1” designates the diatonic dominant with a major ninth, while superscript “2” designates the dominant from the harmonic minor (i.e., with b[♯]6) with a minor ninth. He says that with D² there is only one type of dominant because there exists only one fully diminished seventh chord in the system, which he brackets in the example. However, with D¹, he cites four dominant-seventh possibilities, the diatonic version and then three other “subordinate” versions, based on the inflected scale degrees. In other words, there exist three other diminished triads in the system, as I explained above, each of which represents a subordinate tonality.

The dissonant subdominant and tonic groups are even more interesting. The same distinction for superscript 1 and 2 remains—major ninth and minor ninth above the root, respectively—but since [♯]2 is unalterable, there is only the major ninth for the dissonant tonics (i.e., all superscripts for the T’s are 1, taking [♯]1 as the root of the tonic chord). Both subdominant and tonic groups are further divided into two types: S and Z (subdominant), and T and Q (tonic). In both cases the new letter is used to designate a flatted fifth of the chord, b[♯]6 with the subdominant and b[♯]3 with the tonic (with VI as the root of a submediant functioning as tonic). The “11” that precedes all subdominants and tonics refers to the vertical element of these chords as eleventh chords. If, for instance, the root of the tonic is I and it is a ninth chord, Catoire would use “9T¹.” Because [♯]6 comes in two versions, T can happen with VI or bVI as root, which is designated with a superscript “b1.” With subdominant and tonic groups he lists the scale, below, from which the given chord is derived. The vertical square brackets indicate the notes in the core chord of each harmony, and point to the different diatonic systems contained within each mode of Catoire’s system.

²⁵ *Диссонирующие аккорды мажоро-минорной системы.*

Example 5: Expansion of the Three Chord Groups (Catoire, vol. 1, 61–68)²⁶

a) Dominant Chord Group (p. 61)	b) Subdominant Chord Group (p. 61)	c) Tonic Chord Group (p. 64)
$\begin{bmatrix} \text{VI} \\ \text{IV} \\ \text{II} \\ \text{VII} \\ \text{V} \end{bmatrix} \begin{bmatrix} \text{VI}^{\flat} \\ \text{IV} \\ \text{II} \\ \text{VII} \\ \text{V} \end{bmatrix}$	$\begin{bmatrix} \text{V} \\ \text{III} \\ \text{I} \\ \text{VI} \\ \text{IV} \\ \text{II} \end{bmatrix} \begin{bmatrix} \text{V} \\ \text{III}^{\flat} \\ \text{I} \\ \text{VI} \\ \text{IV} \\ \text{II} \end{bmatrix} \begin{bmatrix} \text{V} \\ \text{III} \\ \text{I} \\ \text{VI}^{\flat} \\ \text{IV} \\ \text{II} \end{bmatrix} \begin{bmatrix} \text{V} \\ \text{III}^{\flat} \\ \text{I} \\ \text{VI}^{\flat} \\ \text{IV} \\ \text{II} \end{bmatrix}$	$\begin{bmatrix} \text{II} \\ \text{VII} \\ \text{V} \\ \text{III} \\ \text{I} \\ \text{VI} \end{bmatrix} \begin{bmatrix} \text{II} \\ \text{VII}^{\flat} \\ \text{V} \\ \text{III} \\ \text{I} \\ \text{VI}^{\flat} \end{bmatrix} \begin{bmatrix} \text{II} \\ \text{VII} \\ \text{V} \\ \text{III}^{\flat} \\ \text{I} \\ \text{VI} \end{bmatrix} \begin{bmatrix} \text{II} \\ \text{VII}^{\flat} \\ \text{V} \\ \text{III}^{\flat} \\ \text{I} \\ \text{VI}^{\flat} \end{bmatrix}$
D ¹ D ²	11S ¹ 11S ² 11Z ¹ 11Z ² Major Mel. Har. Minor Min. Maj.	11T ¹ 11T ^{♭1} 11Q ¹ 11Q ^{♭1} Major Mel. Mel. Minor Maj. Min.

Example 6: Dissonant Z Subdominant Chords (Catoire, vol. 1, 63)

a) b) c) d) e) f)

Z Z¹ [Z¹] 9[Z¹] 11Z¹ [[Z¹]]

Catoire gives examples for all dissonant T, D, and S chords. I have reproduced the examples for the Z chords in Example 6, drawn from the harmonic major scale. Z designates A^b for all chords—if A[♯] were in the chords he would use S. The superscript “1” in Examples 6b–6f designates a major ninth above the root D. Note how, in Examples 6c, 6d, and 6f, Catoire interpolates a bass line with closed noteheads, something like Rameau’s imaginary bass. The square brackets used in those three examples refer to the interpolated bass, that is, the absence of the true root tone. The double square brackets in Example 6f refer to the fact that, for this Z, not only is $\hat{2}$ absent as the root tone, but $\hat{4}$ is also absent. In Example 6f, because the first dominant resolution resolves to a 6/4 chord position, the example continues to the first inversion tonic at the end of the example. The diagonal lines in each example refer to the resolution of dissonances.

²⁶This example appears in slightly altered form in Carpenter 1988, 588.

Finally, the 9 that precedes Z in Example 6d refers to the fact that, above the actual root of the chord, F, there is a ninth, G, while the 11 that precedes Z in Example 6e refers to the fact that there is an eleventh above the actual root and bass note, D.

In order to show how Catoire analyzes music in his textbook I have reproduced two short examples. In Example 7 he analyzes the beginning of the second movement of Beethoven's Piano Sonata No. 31. The two dominants analyzed feature his markings. The interpolated bass, C2–G2–C2 (with downward stems) is shown in square brackets in mm. 2–4 in the score. The first analyzed dominant, over B^b2 in the bass on the second beat of m. 2, is notated with “[D²].” The “D” is uppercase because the seventh, B^b, is in the chord—again, if the dominant were a consonant triad Catoire would use a lowercase “d”—and there is a superscript “2” because the minor ninth of the C dominant ninth, D^b, is also in the chord. Finally, the “D” is in square brackets because the root of the dominant, C, is not in the actual music. The second analyzed dominant is exactly like the first, with one exception—it is a secondary dominant, tonicizing the C major dominant on the downbeat of m. 4, and Catoire therefore uses the roman numeral “II” beside it.

Example 8 shows Catoire's analysis for a four-measure excerpt from Schumann's *Carnaval*.²⁷ The excerpt begins with the submediant, which he designates with roman numeral VI.²⁸ The dominant in m. 3 is an uppercase “D” since it has the chordal seventh. The chord in m. 2 is intriguing. Once again he uses an interpolated bass, E^b2, below the chord in question. Catoire calls the “[U²]” chord a “fully diminished seventh chord with a lowered third” (Vol. 1, 94). Thus this chord is a primary dominant with a minor ninth above the root E^b2 (therefore the superscript “2”), and the “2” preceding the harmony refers to the inversion of the seventh chord. This is confusing since this is also where he places “9” and “11” for ninth and eleventh chords. Catoire makes no mention of the strong chromatic ascent from A^b4 to C5 in this excerpt. This said, there are no such linear connections in Catoire's textbook—his was a rigid, yet rigorous account of verticalities in contemporary chromatic music.

²⁷ I was unable to find where this excerpt happens exactly in Schumann's *Carnaval*, op. 9.

²⁸ Catoire does not distinguish between upper and lowercase roman numerals with respect to mode.

Example 7: Catoire, Analysis of Beethoven, Op. 110, II, beginning (Catoire, vol. 1, 86)

The musical score for Example 7 shows the beginning of the second movement of Beethoven's Op. 110. It is in 2/4 time, key of D minor, and marked piano (p). The bass line includes harmonic analysis labels: $[D^2]$ and $[D^2]_{II}$.

Example 8: Catoire, Analysis of Schumann, “Carnaval” (Catoire, vol. 1, 94)

The musical score for Example 8 shows the beginning of Schumann's "Carnaval". It is in 3/2 time, key of D minor, and marked piano (p). The bass line includes harmonic analysis labels: VI, $2[U^2]$, and D.

Catoire's significance in the history of Russian music theory remains underexplored, and his textbook, which he used for his harmony classes at the Moscow Conservatory, remains ripe for exploration. He had several students at the conservatory who would go on to write the most important and enduring harmony textbook in Soviet Russia, the *Учебник гармонии* (Harmony textbook), usually called the *Бригадный учебник* ("Brigade" textbook), which my colleague Ellen Bakulina unpacks in her contribution to this issue of *Theoria*. The brigade authors—Joseph Dubovsky (1892–1969), Sergei Evseev (1894–1956), Vladimir Sokolov (1897–1950), and Igor Sposobin (1900–1954)—all studied with Catoire and, believe it or not, this harmony textbook is still widely used in the Russian Federation today.²⁹ It was first published in two volumes in 1934 and 1936 as *Практический курс гармонии* (A practical course of harmony),

²⁹ The textbook has even had a storied history in China (see Cheong and Hong 2018).

but the second edition (Dubovsky et al., 1939), the *Учебник гармонии*, is what remained. It is currently in its fifth edition, published in 2016.

Yuri Tiulin and the Leningrad School of Music Theory

In order to understand the work of Yuri Tiulin (1893–1978) one must put him into the context of Leningrad, where he taught for much of his career, and the so-called “Leningrad School of Music Theory.”³⁰ In her article on the subject, Tatiana Bershchadskaya cites three main figures—Boris Asafiev, Yuri Tiulin, and Christopher Kushnarev—as the instigators of the Leningrad School (Bershchadskaya 2013, 9). However, the true lineage of this school can be traced directly to Boleslav Yavorsky (1877–1942) and his “theory of *lādovy* rhythm.”³¹ But Yavorsky and his theories, on which Asafiev and then Tiulin drew extensively, did not develop in a vacuum. Rather, they arose in significant part as a counterbalance to the encroaching harmonic functionalism of Riemann in Russia in the early to mid-twentieth century as represented by Catoire and his Moscow cohort. In large part, Tiulin bridged this divide between Riemann/Catoire on the one hand and Yavorsky/Asafiev on the other. Though I focus primarily on his use of harmonic functionalism, Tiulin’s textbook exhibits elements of both schools of thought. As Ellon Carpenter notes, “Tiulin’s theories of function were stimulated by those of Riemann, and his theories of mode by those of Yavorsky” (1988, 1111).³²

Asafiev was clearly inspired by Yavorsky, while at the same time he took a hard line against the intruding harmonic functionalism represented by Riemann and his proponents in Russia:

Among theorists it was the Russian musician-thinker Boleslav Yavorsky who undertook a deep analysis of “tritonality” and discovered the meaning of its intonational purview in contemporary music.

³⁰ For more on this school see Ewell 2020b.

³¹ For more on this theory see Ewell 2020a. “*Lād*” [Russian *лад*] is a term that, in the nineteenth century meant first “tonality” and then “mode.” In Tiulin’s work he usually means the former, though he occasionally uses it to mean the latter. I will specify, below, which is which when the term is used.

³² Carpenter discusses Tiulin in 1988, 1102–1139. This material is also largely contained in Carpenter (1983) 2009, 344–359.

On the other hand, Riemann's system of "functional harmony," which has slavishly subordinated the minds of many theorists, subjugates the composer's hearing and consciousness with its conservative, mechanical "predetermination." This system is the sad legacy of the so-called "general bass," figured bass, i.e., the teaching of harmony born of the practice of organ and piano accompaniment, some kind of "accompaniment school."³³ (Asafiev [1930 and 1947] 1971, 2:243–44)

This quotation reveals the genesis of the Leningrad school of music theory. Asafiev goes on to further rebuke Riemann and his denial of the "physiological" and "intonational" aspect of music (245–246).

Tiulin's career began with his *Учение о гармонии* (Study of harmony) (Tiulin [1937, 1939] 1966). His *Учебник гармонии* (Harmony textbook), which he cowrote with Nikolai Privano, represents Leningrad's answer to the much more famous Muscovite brigade textbook (see Tiulin and Privano [1957, 1959, 1961] 1964). Tiulin and Privano's textbook has a Riemannian angle—T, D, and S, are used prominently early on—but it is much more muted when compared to the brigade textbook. Strikingly, Tiulin and Privano's harmonic analyses of excerpts are done almost entirely with roman numerals and not with the three Riemannian descriptors as they are in the brigade textbook, which shows the authors' penchant for *Stufentheorie* over *Funktionstheorie*.

Tiulin undertook an in-depth analysis of Russian folk music, much like Yavorsky before him, in order to form his theories. He studied the role of strong "stable" tones in the "formation of the *lād*" (*ладообразование*) in this music within its diatonic framework. Ultimately, he links his findings to acoustical-physiological rules, and then the "psychological rules of comprehension" (Tiulin [1937, 1939] 1966, 5).³⁴ This indicates Tiulin's indebtedness to Yavorsky, which suggests that, even though he praises both in the text, he ultimately sides with Yavorsky in forming his harmonic theories, which is why Tiulin is so strongly linked with the Leningrad School.

³³ *Из теоретиков глубокий анализ "тритонности" и раскрытие значения этой интонационной сферы в современной музыке дал русский музыкант-мыслитель Б.Л. Яворский.*

Наоборот, рабски подчинившая себе умы многих теоретиков римановская система "функциональной гармонии" закрепощает слух и сознание композиторов своей консервативной механической "предустановленностью". Эта система является печальным наследием так называемого "генерал-баса", цифрованного баса, т.е. учения о гармонии, рождавшегося из практики органного и клавирного сопровождения, своего рода аккомпаниаторства.

³⁴ This information comes from the Preface—written by Nikolai Privano, Tiulin's coauthor for his harmony textbook—to the 1966 edition.

In fact, there is only one point in *Study of Harmony* that Tiulin takes Yavorsky to task. Yavorsky's "theory of *lādovy* rhythm" was based almost exclusively on tritone resolutions. In a lengthy footnote, Tiulin finds fault with this by saying, correctly, that Yavorsky insists on this resolution even when there are no tritones in the music, such as with a "very simple folk song" (Tiulin [1937, 1939] 1966, 127n1). Tiulin refers to this as "theoretical dogmatism," a significant slur against Yavorsky in the 1930s when Yavorsky was the most famous theorist in Russia.³⁵ Aside from this one fault, Tiulin relies heavily on Yavorsky and uses his terminology frequently: *lād* (and all of its derivations), conjunction, gravitation, stability, and the emphasis on psychological and physiological elements of music, among other things.

In *Study of Harmony* Tiulin also mentions Riemann frequently throughout the entire textbook.³⁶ Tiulin faults Riemann for his lack of understanding of the psychological aspect of music, echoing Asafiev's criticism above. In fact, Asafiev, the older and more famous of the two, may have drawn his criticism of Riemann from Tiulin, whose work predates Asafiev's by some ten years. Under a heading entitled "The Meaning of Mutable Functions,"³⁷ about which more below, Tiulin writes about the beauty of the psychological perceptive elements of these functions, citing Chopin's Prelude no. 25, op. 45. Tiulin then says about Riemann's functionalism:

Thus it becomes clear that the theory of harmonic and melodic variable functions is based precisely on the psychological factor of musical perception. This distinguishes it from Riemann's function theory, which considers only the basic logical connections

³⁵ One wonders whether this particular footnote may have been added after Yavorsky's death in 1942.

³⁶ Tiulin even makes mention of Heinrich Schenker's 1906 *Harmonielehre* at one point, emphasizing his discussion of "tonicization." See Tiulin (1937, 1939) 1966, 154n1. This is significant since Tiulin began this work when Schenker was still alive. He may have learned about *Harmonielehre* from Mikhail Ivanov-Boretsky (1874–1936), an important historian and composer who, like Catoire, studied composition with Rimsky-Korsakov in the late nineteenth century. To Ivanov-Boretsky belongs the first mention in the Russian literature, in 1931, to Heinrich Schenker—he also cites "tonicization" in Schenker's work. As the general editor of a translation into Russian of Lucien Chevailler's *Les théories harmoniques*, Ivanov-Boretsky said that Schenker, in his *Harmonielehre*, "establishes the possibility of so-called tonicization, that is, the striving of the scale degrees of the *lād* [i.e., tonality] to transform into their own tonics with their own dominants and subdominants" (*устанавливает возможность так наз. тоникализации, т. е. стремления ступеней лада превращаться в самостоятельные тоники с собственными доминантами и судоминантами*) (Chevailler 1931, 152).

³⁷ *Значение переменных функций.*

of chords, disregarding psychological perception. From Riemann's point of view these connections do not find explanation for the correlation of chords of the subordinate scale degrees.³⁸

Thus it is possible to include Tiulin in the general move away from Riemann and functionalism, which is represented by the so-called “Leningrad (now St. Petersburg) School of Music Theory.”³⁹

So what exactly are “variable functions” (*переменные функции*) to Tiulin?⁴⁰ He explains this in the Chapter “Fundamental and Variable Functions of Tones in a Tonality.”⁴¹ After establishing the fifth relation as axiomatic, Tiulin shows how this establishes the T, D, and S of any tonality. He speaks of harmonic “gravitation” ([1937, 1939] 1966, 151; this term is from Yavorsky), and how T, D, and S occupy a special place as fundamental harmonies and establish keys through authentic cadences. But when another key is tonicized, a “variation of functions” (*перемена функций*) occurs. Further a “lādovy conflict” occurs, which requires resolution further on (152–153). Notably, Tiulin faults Riemann for his denial of these variable functions:

Riemann's functional theory, which occupies firm ground in German musicology, is limited to its fundamental functional cell, ignoring the other functional connections of the other tones of the lād [tonality]. This is understandable: in his study Riemann neglects the variable functions while it is precisely these functions that play a large role

³⁸ Таким образом, становится ясным, что теория гармонических и мелодических переменных функций основана на учете именно психологического фактора музыкального восприятия. Этим она и отличается от римановской функциональной теории, принимающей во внимание только основные логические связи аккордов, в отрыве от психологии восприятия. С точки зрения Римана не находят своего объяснения соотношения аккордов побочных ступеней.

³⁹ The undisputed leader of the Leningrad School is currently Tatiana Bershchadskaia, who studied with Tiulin at the Leningrad Conservatory. For more on her see Ewell 2020b. Her *Лекции по гармонии* (Lectures in harmony) (Bershchadskaia [1978, 1985] 2003) is, in many ways, an extension of Tiulin's *Study of Harmony*.

⁴⁰ The Russian here for “variable” is *переменный*, which is also translated as “mutable,” as in “mutable lāds” (*переменные лады*). Carpenter calls the mutable lāds “variable” lāds, thus keeping the same translation for *переменный*. For more on mutable lāds, see Bakulina 2014 and 2015 (Bakulina calls them mutable “modes”).

⁴¹ Основные и переменные функции тонов лада.

in artistic practice, defining the connection between the *lādovy* [tonal] periphery and the *lādovy* [tonal] center.⁴² ([1937, 1939] 1966, 154–155)

Example 9 shows fifth relations in two of Tiulin’s main harmony texts, the first from 1937, the second from 1957 (this is from the textbook that he coauthored with Nikolai Privano). In the first instance (Example 9a), he includes this discussion in the chapter on “Triads of the Tonality,” in which he immediately discusses triads whose roots are related by second (no common tones), fifth (one common tone), and third (two common tones). This happens late in the book. In the second instance, however, this chart happens early on, in the chapter “Variable Functions of Chords.” It seems that over 20 years Tiulin had refined his beliefs about harmonic functionalism and felt he could present it to students more or less from the beginning of their studies. Note in Example 7 the differences between the two editions. Main and subordinate “centers” became “groups,” and VI and III became submediant and mediant. Also, instead of showing common tones between adjacent chords (Example 9a), Tiulin shows harmonic motion to the tonic or its relative minor, the submediant (Example 9b). Finally, insofar as these two examples are for the major mode, the tones of C major are rendered with open noteheads while all others are rendered with closed. Importantly, Tiulin combines Riemannian functions (T, D, and S) with roman numerals, an admixture of function and step theories.

Example 10a shows Tiulin’s tertian diagram of functional properties for all triads of a tonality, in this case C major.⁴³ In setting up this example Tiulin says, “the tertian succession is the best example in terms of *lādovy* relationships since it completely reveals the *lādovy* relations of triads” ([1937, 1939] 1966, 175).⁴⁴ I hesitate to insert “tonality” after “*lād*” here since the latter, by 1937, already meant so much more than simply “tonality” in Russia. What Tiulin is showing in Example 10a is a tonality as *lād*, with a *lādovy* center, which meets at the “axis of equilibrium,” and a *lādovy* “periphery” (*периферия*), represented by the closed-note extremes of the example. The immediate

⁴² *Функциональная теория Римана, занявшая прочное положение в германской музыкальной науке, ограничивается основной функциональной ячейкой, игнорируя функциональные связи остальных тонов лада. Это и понятно: Риман в своем учении упускает из вида переменные функции, между тем как именно эти функции играют большую роль в художественной практике, определяя связь ладовой периферии с ладовым центром.*

⁴³ This example is also contained in Tiulin/Privano, 28–29.

⁴⁴ *Наиболее показательным в ладовом отношении является терцовый ряд, так как он полностью вскрывает ладовое родство трезвучий.*

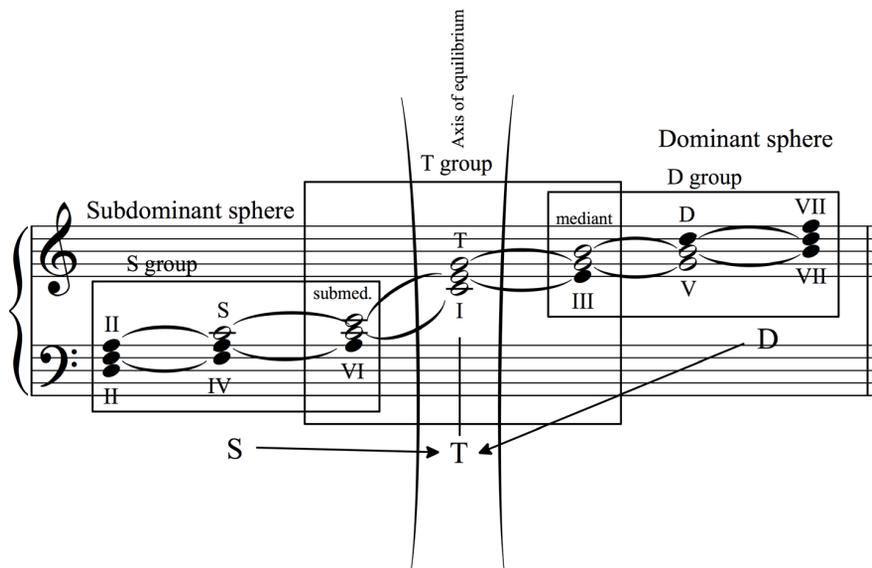
square brackets around the axis represent D and S with common tones to T, while the square brackets further out represent the entirety of D and S, which both lead back to T with arrows. Tiulin says that the further away from the T center, the less stable the triad, which “strengthens its functionality” (176) in its drive toward the tonic. Example 10b shows the complete tertian aspect of this arrangement, as a d^{15} chord, in which a D-minor triad (the bottom three notes) moves to a B-diminished triad (the top three notes), to a C-major triad (the middle three notes).

Example 9: Fifth Relationships of Triads in the Major Mode, in a) Tiulin (1937, 1939) 1966, 174, and b) Tiulin and Privano (1957, 1959, 1961) 1964, 33

Example 10a and 10b: Tertian Diagram of the Functional Properties of All Triads in a Tonality (Tiulin [1937, 1939] 1966, 175)

a.

b.

Example 11: The Three Functional Groups (Tiulin [1937, 1939] 1966, 177)⁴⁵

Finally, Tiulin includes the most complete example, my Example 11, under the heading “Functional Groups of Triads” ([1937, 1939] 1966, 176–177).⁴⁶ Note immediately, once again, the admixture of function theory and step theory. Tiulin discusses the role of the mediant as both T and D and the submediant as both T and S, and suggests the designations TD and TS, respectively. Taking the three boxes in Example 11 as a Venn diagram, Tiulin usefully places these mediants into both “spheres” of influence. Further, because of their distance from the T axis of equilibrium, II and VII are “extreme” and “unstable” elements, and cannot relate to T directly. After this discussion he shows my Example 11 as “the three functional groups T, D, and S.”

Unlike Catoire, Tiulin does no direct harmonic analysis in his *Study*. There are many musical examples by composers such as Bach, Beethoven, Chopin, Mozart, Mussorgsky, and Schubert, but no annotated analyses. Rather, Tiulin discusses the

⁴⁵ A similar graph to this appears in Carpenter 1988, 1133. She gets her version from Tiulin/Privano, 30.

⁴⁶ *Функциональные группы трезвучий.*

pieces in the body of the text.⁴⁷ Thus his is a more abstract text, in this regard, than Catoire's. In the textbook coauthored with Privano, there are many written analyses, yet none are from the literature—all are exercises and models composed by the authors. Further, virtually all harmonic analyses feature roman numerals. Only on rare occasions, such as in the section on secondary dominants, can one see the use of “D” and “S,” and sparingly at that. Still, the impact of Tiulin on music theory in Russia, and especially in St. Petersburg, cannot be overstated.

Conclusion

The history of harmonic functionalism in Russia is, in fact, remarkably complex. The intense pitched battles over nationalism in the nineteenth century pitted, to a large extent, those who would look to the West for inspiration against those who would look inward, the nationalists. It goes without saying that, to draw inspiration from the German Hugo Riemann represented a look to the West. And that Catoire, who was ethnically French, brought the system to Russia from Germany further complicated matters. For this reason, a homegrown version of music theory was born in Russia in the twentieth century, represented primarily by Yavorsky. His ideas spawned an entire school of music theory, the Leningrad/St. Petersburg school, that remains underexplored outside of Russia. Many significant works by subsequent authors such as Boris Asafiev, Christopher Kushnarev, Tiulin, and Tatiana Bershchadskaia pushed back against Riemannian influence in Russia, and what they in St. Petersburg ultimately labelled the “Moscow School” of music theory, which was represented by Catoire's four students who wrote the “brigade” textbook and, afterward, by Yuri Kholopov, who studied music theory for four years at the Moscow Conservatory with its most famous coauthor, Igor Sposobin. Ultimately, both schools of music theory rely quite heavily on aspects of both step theory and function theory. It could be said that American theory does as well. Most current U.S. music theory textbooks, though steeped primarily in Schenkerian thought, have prominent sections outlining the three harmonic functions T, D, and S. Thus even American textbooks can be said to be hybrids of these two systems. But this dual borrowing is more prominent in Russia—their functionalism grew out of a step theory that never really left Russian music theoretical thought, from the nineteenth century until today.

⁴⁷ Exactly the same thing can be said of Bershchadskaia's *Lectures in Harmony*.

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